You can write to Her Malesty at the following address Her Majesty The Queen Buckingham Palace London SW1A 1AA

If you wish to write a formal letter, you can open with 'Madam' and close the letter with the form 'I have the honour to be, Madam, Your Majesty's humble and obedient servant?

This traditional approach is by no means obligatory. You shou feel free to write in whatever style you feel comfortable.